

## Hot phonons and electrons in semiconductors

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A general approach to study of energy interaction between hot electrons and phonons is presented. Basing on kinetic equations for both subsystems of these quasiparticles we suggest the most convenient models for the theoretical investigation of hot electrons as well as of hot phonons. All of these models are based on possibility of obtaining the symmetric part of nonequilibrium distribution function in the form of the Dirac function (for electrons) or the Planck function (for phonons) with own nonequilibrium local temperatures. General equations are obtained for these temperatures.

*Keywords:* Boltzmann equations; Hot electrons; Hot phonons

### 1. Introduction.

Question about transport of nonequilibrium carriers many decades is one of the central problems of Solid State Physics [1]. At the same time overwhelming majority of works is devoted to the transport of nonequilibrium charge carriers [2]. Nevertheless, as it is well known, a phonon subsystem can be easy to turn out from the equilibrium thermodynamic state too. In these cases fluxes of different nature occur in this subsystem, and these fluxes essentially result in the transport of charge carriers. Generally, the interaction between nonequilibrium charge carriers and nonequilibrium phonons is carried out with transferring of energy as well as with transferring of directed momentum [3, 4].

It is well known that different external agents can readily produce in a semiconductor an essential increase in the average energy of charge carriers and phonons (energy nonequilibrium) [4], which can be most conveniently described in terms of heating of the charge carrier and phonon gas (i.e. increase of their respective temperatures  $T_e$  and  $T_p$ ). Since heating is known to exert a pronounced influence on transport phenomena in semiconductors, being largely dependent on the character of transport itself [4], it is quite understandable that the heating of electrons and phonons should be essentially interdependent. Note that the possibility of phonon deviation from equilibrium was first considered by R. Peierls [5] (see also [6]).

The heat generated by hot carriers can be harvested to drive a wide range of physical and chemical processes. Their kinetic energy can be used to harvest solar energy or create sensitive photodetectors and spectrometers. Photojected

charges can also be used to electrically dope two-dimensional materials [7].

Hot (nonequilibrium) phonon effects on electron transport in rectangular GaAs/AlAs quantum wires have been investigated by a self-consistent Monte Carlo simulation [8]. Authors have demonstrated that at room temperature hot optical phonons lead to a significant increase in electron drift velocity.

Steady state and modulated heat conduction in layered systems predicted by the analytical solution of the phonon Boltzmann transport equation was presented in [9] and has widely been used as the theoretical framework for the development of photoacoustic and photothermal techniques.

In work [10] the electron and phonon temperature distribution functions in semiconductors are calculated. The electron and phonon temperature distributions in the sample are given as a function of both, time and position valid for a wide range of the modulation frequency of the incident light.

The term “hot carrier injection” usually refers to the effect in MOSFETs, where a carrier is injected from the conducting channel in the silicon substrate to the gate dielectric, which usually is made of silicon dioxide ( $\text{SiO}_2$ ). [11]. In some semiconductor devices, the energy dissipated by hot electron phonons represents an inefficiency as energy is lost as heat. For instance, some solar cells rely on the photovoltaic properties of semiconductors to convert light to electricity. In such cells, the hot electron effect is the reason that a portion of the light energy is lost to heat rather than converted to electricity [12].

The physical peculiarities of the thermoelectric cooling phenomenon by an electric current in p-n structures of thermally thick structure in the linear approximation are investigated in [13].

It is shown the possibility of the realization of an exotic distribution of the temperatures of electrons, holes, and phonons.

The aim of the present paper is to present a short review of rigorous kinetic approach to energy interaction between nonequilibrium charge carriers and nonequilibrium phonons.

## 2. Kinetics of nonequilibrium electrons and phonons in semiconductors

Let us consider for simplicity an unipolar semiconductor (for example with electron type conductivity) with a quadratic and isotropic dispersion law for charge carriers,

$$\varepsilon = \frac{\vec{p}^2}{2m} \quad (1)$$

where  $\varepsilon$ ,  $\vec{p}$ , and  $m$  are the electron energy, quasimomentum, and the effective mass. The acoustic phonons dispersion law is

$$\omega = \frac{\vec{s}\vec{q}}{\hbar}. \quad (2)$$

Here  $\omega$  is the acoustic phonon frequency,  $\vec{s}$  is the sound velocity in a material, and  $\vec{q}$  is the phonon quasimomentum. Kinetics of electron and acoustic phonons (we will write "phonons" later everywhere supposing that optic phonons do not take part in a considered problems) in a heating electric field  $\vec{E}$  and a temperature field is described by set of the following coupled Boltzmann equations [4, 14],

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{r}} + e\vec{E} \frac{\partial f}{\partial \vec{p}} = S_{ep} + S_{ed} + S_{ee} \quad (3)$$

$$\frac{\partial N}{\partial t} + \vec{s} \frac{\partial N}{\partial \vec{r}} = S_{pe} + S_{pd} + S_{pp}. \quad (4)$$

Here  $f(\vec{r}, \vec{p}, t)$  and  $N(\vec{r}, \vec{q}, t)$  are the nonequilibrium electron and phonon distribution functions;  $\vec{v}$  is the electron velocity;  $e$  is the electron charge;  $S_{ep}$ ,  $S_{ed}$ ,  $S_{ee}$ ,  $S_{pe}$ ,  $S_{pd}$  and  $S_{pp}$  are the electron-phonon, electron-defect, electron-electron, phonon-electron, phonon-defect, and phonon-phonon collision integral accordingly.

We suppose that all conditions ensuring applicability of the kinetic equation method are to be satisfied (see, Ref.[15]). We use the energy system units throughout the paper, so temperature has the energy dimension. As was shown in Ref.[16] the electron-acoustic phonon scattering is quasielastic at temperatures just above

$1 K$  ( $\hbar\omega \ll \bar{\varepsilon}$ , where  $\bar{\varepsilon}$  is the electron average energy). Electron scattering on the defects always is elastic [16].

The made assumption about the quasielastic scattering on the phonons gives possibility to represent the electron distribution function in diffusion approximation [4, 16],

$$f(\vec{r}, \vec{p}, t) = f_0(\vec{r}, \varepsilon, t) - \vec{V}(\vec{r}, \varepsilon, t) \vec{p} \frac{\partial f_0(\vec{r}, \varepsilon, t)}{\partial \varepsilon}, \quad (5)$$

where  $f_0(\vec{r}, \varepsilon, t)$  is the isotropic part of the distribution function,  $\vec{V}(\vec{r}, \varepsilon, t) \vec{p} \frac{\partial f_0(\vec{r}, \varepsilon, t)}{\partial \varepsilon}$  is the anisotropic part of the distribution function which is much less than  $f_0(\vec{r}, \varepsilon, t)$ , and  $\vec{V}(\vec{r}, \varepsilon, t)$  is some function depending on coordinates, energy and time.

Using the same approximation the phonon distribution function can be written in the following form,

$$N(\vec{r}, \vec{q}, t) = N_0(\vec{r}, \hbar\omega, t) - \vec{u}(\vec{r}, \hbar\omega, t) \vec{q} \frac{\partial N_0(\vec{r}, \hbar\omega, t)}{\partial (\hbar\omega)}, \quad (6)$$

Where  $N_0(\vec{r}, \hbar\omega, t)$  is the isotropic part of the phonon distribution function,  $\vec{u}(\vec{r}, \hbar\omega, t) \vec{q} \frac{\partial N_0(\vec{r}, \hbar\omega, t)}{\partial (\hbar\omega)}$  is the anisotropic part of the phonon distribution function which is also much less than  $N_0(\vec{r}, \hbar\omega, t)$ ,  $\vec{u}(\vec{r}, \hbar\omega, t)$  is some function depending on coordinates, phonon energy and time.

In this paper later we will consider only the stationary case.

It is well known that the electron distribution function form essentially depends on ratio between the electron-electron collisions rate  $\nu_{ee}(\varepsilon)$ , the electron momentum relaxation rate  $\nu(\varepsilon)$ , and the electron energy relaxation rate  $\nu_\varepsilon(\varepsilon)$  [17]. We will suppose that these rates satisfy the following correlations in this work,

$$\nu(\varepsilon) \gg \nu_{ee}(\varepsilon) \gg \nu_\varepsilon(\varepsilon). \quad (7)$$

Approximation (7) is often called the partial or energy control [18]. In this case the electron subsystem can be described by means of the temperature approximation with the isotropic part of distribution function like the Fermi-Dirac (Maxwell) distribution function [18],

$$f_0(\vec{r}, \varepsilon) = \frac{1}{1 + e^{\frac{\varepsilon - \mu(r, T_e(r))}{T_e(r)}}}, \quad (8)$$

where  $\mu[\vec{r}, T(\vec{r})]$  is the electron chemical potential,  $T_e(\vec{r})$  is the electron temperature.

The brief proof of the stated above consists in the following. The largest term in Eq.(3) is the electron-electron collision integral which is the order of  $v_{ee}f_0$  [15]. By this reason the symmetric part of the distribution function  $f_0$

can be represented as a series in powers of  $\frac{v_\varepsilon}{v_{ee}}$ . In the zero

approximation with respect to this parameter the kinetic equation (3) reduces to  $S_{ee}(f_0) = 0$ , and it is clear that solution of this equation is the function (8) since only the function like the Fermi-Dirac function turns into zero the collision integral  $S_{ee}(f_0)$ .

Later we will consider only nondegenerate semiconductors, and will assume that the Debye length

$d = \sqrt{\frac{\varepsilon_0 T_e}{4\pi e^2 n}}$  is the smallest characteristic length of the

problem,  $\varepsilon_0$  is the permittivity,  $n$  is the electron concentration. The later supposing means that a plasma in semiconductor is quasineutral, and the concentration  $n$  does not depend on coordinates in the case of unipolar semiconductors. In this case the electron concentration is equal to its equilibrium value (we take off from consideration processes such as impact ionization, recombination rate change in strong electric fields, etc). Now let us to turn to the phonon subsystem, and first of all examine the collision integral  $S_{pe}$  which can be written as [4],

$$S_{pe} = \int d^3 \vec{p} W(\vec{q}) \{ f(\vec{p} + \vec{q}) [1 - f(\vec{p})] [N(\vec{q}) + 1] - f(\vec{p}) [1 - f(\vec{p} + \vec{q})] N(\vec{q}) \} \times \delta[\varepsilon(\vec{p} + \vec{q}) - \varepsilon(\vec{p}) - \hbar\omega(q)] \quad (9)$$

Here  $W(\vec{q})$  is a value which is proportional to a matrix element which corresponds to electron transition from the state with the quasimomentum  $\vec{p}$  to the state  $\vec{p} + \vec{q}$  due to collision with phonon having the quasimomentum  $\vec{q}$ .

To transform the collision integral (9) to more convenient form, we shall use the identity being true for the Fermi distribution function (8),

$$f_0(\varepsilon + \hbar\omega) [1 - f_0(\varepsilon)] = N_{T_e} [f_0(\varepsilon) - f_0(\varepsilon - \hbar\omega)], \quad (10)$$

where

$$N_{T_e} = \frac{1}{e^{\frac{\hbar\omega}{T_e}} - 1} \quad (11)$$

is the Planck distribution function with the temperature  $T_e$ .

Substituting (10) into (9) and integrating over the solid angle (the axis oz is directed along the vector  $\vec{q}$ ), we can obtain that

$$S_{pe} = (N_{T_e} - N) \frac{2\pi m^2 W(\vec{q})}{q} \int_{\varepsilon_{\min}}^{\infty} d\varepsilon [f_0(\varepsilon) - f_0(\varepsilon + \hbar\omega)] + \left( N + \frac{1}{2} \right) \frac{2\pi m^2 W(\vec{q})}{q} \bar{q} \int_{\varepsilon_{\min}}^{\infty} d\varepsilon V(\varepsilon) \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \quad (12)$$

Here

$$\varepsilon_{\min} = \frac{1}{2m} \left( \frac{1}{2} q - \frac{m\hbar\omega}{q} \right)^2 \approx \varepsilon \left( \frac{q}{2} \right). \quad (13)$$

The first item in Eq.(12) has a very clear sense, only those phonons interact with electrons (with energy exchange) which have the temperature different from the electron temperature  $T_e$ . Thus, the factor at the difference  $N_{T_e} - N$  can be interpreted as the phonon-electron relaxation rate  $v_{pe}(q)$ . Integrating over  $\varepsilon$  yields

$$v_{pe}(q) = \left[ \frac{2\pi m^2 W(q)}{q} \right] T_e \ln \left[ 1 + \frac{f_0\left(\frac{q}{2}\right)}{N_{T_e}} \right]. \quad (14)$$

Since  $f_0 \ll 1$  for nondegenerate electron gas and  $\bar{\varepsilon} \propto T_e \gg \hbar\omega$  at quasielastic scattering, then

$$N_{T_e} \approx \frac{T_e}{\hbar\omega} \gg 1 \text{ and } \ln \left[ 1 + \frac{f_0\left(\frac{q}{2}\right)}{N_{T_e}} \right] \approx \frac{\hbar\omega}{T_e} f_0\left(\frac{q}{2}\right).$$

Thus,

$$v_{pe}(q) = 2\pi m^2 W(q) \frac{\hbar\omega}{q} f_0\left(\frac{q}{2}\right) \quad (15)$$

and the phonon-electron collision integral takes the form,

$$S_{pe} = v_{pe} (N_{T_e} - N) + 2\pi m^2 W(q) \left( N + \frac{1}{2} \right) \frac{\bar{q}}{q} \int_{\varepsilon\left(\frac{q}{2}\right)}^{\infty} d\varepsilon V(\varepsilon) \left( -\frac{\partial f_0}{\partial \varepsilon} \right) \quad (16)$$

Eq. (13) determines the lowest energy for electron being able to interact with a phonon with the quasimomentum  $q$ . The corresponding lowest electron momentum is

$$p_{\min} = \left| \frac{q}{2} - \frac{m\hbar\omega}{q} \right|.$$

Since the momentums of electrons and phonons are of the same order at quasielastic scattering, the second item in the right-hand side is the order of  $q \frac{S}{v_T}$  ( $v_T$  is the electron

thermal velocity), and we can neglect it because  $\frac{S}{v_T} \ll 1$ .

As can be seen from Eq.(15), electrons interact preferably with those phonons which have the momentum  $q < 2\bar{p}$ , where  $\bar{p}$  is average electron momentum ( $\bar{p} \propto \sqrt{2mT_e}$ ).

These phonons are known as the long-wave phonons (LW). The phonons with momentum  $q > 2\bar{p}$  are called the short-wave phonons (SW).

Naturally, contribution of phonons to kinetic phenomena essentially depends on its number. Let  $T_p$  is the characteristic phonon temperature (see below in detail). In

this case the momentum  $q_{T_p} \propto \frac{T_p}{S}$  can be account as the top limit of the phase space which the phonons occupy (number of phonons with momentum  $q > q_{T_p}$  is exponentially small, and we can neglect them). It is clear that the number of LW-phonons and SW-phonons depends on the correlation between momentums  $2\bar{p}$  and  $q_{T_p}$ . Later, we will consider two following limiting cases:

1. LW-phonons occupy a small phase space volume,  $2\bar{p} \ll q_{T_p}$ . (17)

This equation leads to the following limitation of the electron temperature,

$$T_e \ll T_p \frac{T_p}{8ms^2}. \quad (18)$$

Let us note that Eq.(18) does not contradict the strong electron heating  $T_e \gg T_p$ .

2. LW-phonons occupy a large phase space volume,

$$2\bar{p} \gg q_{T_p}. \quad (19)$$

In this case electrons interact with all phonons, and the electron temperature satisfy the condition  $T_e \gg T_p \frac{T_p}{8ms^2}$

It's easy to see from Eq. (14) that the rate  $v_{pe}(q)$  rapidly decreases for  $q > 2\bar{p}$  tending to zero. By this reason, the correlation

$$v_{pp} \gg v_{pe} \quad (\text{at } q > 2\bar{p}) \quad (20)$$

always takes place for SW-phonons. Here  $v_{pp}$  is the SW-phonons-LW-phonons rate interaction.

At the same time both correlations

$$v_{pp} \gg v_{pe} \quad (\text{at } q < 2\bar{p}) \quad (21)$$

and

$$v_{pp} \ll v_{pe} \quad (\text{at } q < 2\bar{p}) \quad (22)$$

can take place for LW-phonons.

Since the phonon-phonon collision rate depends on phonon temperature, Eqs.(21) and (22) determine the temperature region for phonons and electrons respectively at which the inequalities (21) and (22) are hold true. Thus, for example Eq.(22) holds at  $T < 50K$  for  $n-Ge$  (deformation acoustic electron-phonon scattering) and  $n-GaAs$  (piezoelectric acoustic electron-phonon scattering) at the electron concentration  $n \approx 10^{14} \text{ cm}^{-3}$  [19].

### 3. Strong phonon-phonon interaction

If the condition  $v_{pp} \gg v_{pe}$  holds true for all  $q$  then  $N_0$  is the Planck distribution function with the temperature  $T_p$ ,

$$N_0 = N_{T_p} = \frac{1}{e^{\frac{\hbar\omega}{T_p}} - 1} \quad (23)$$

In this case one could obtain by usual technique [3] the energy balance equation for phonon subsystem,

$$\text{div} \bar{Q}_p = P_{pe}. \quad (24)$$

Here

$$P_{pe} = \frac{1}{(2\pi\hbar)^3} \int \hbar\omega v_{pe} (N_{T_e} - N_{T_p}) d^3\vec{q} \quad (25)$$

is the energy flux from the electron to phonon subsystem;

$$\bar{Q}_p = \frac{1}{(2\pi\hbar)^3} \int q\hbar\omega \left( -\frac{\partial N_{T_p}}{\partial(\hbar\omega)} \right) \vec{u} \frac{S}{3} d^3\vec{q} \quad (26)$$

is the heat flux in the phonon subsystem.

It follows from definition of the value  $P_{pe}$  that

$$P_{pe} \propto nT v_{\varepsilon}, \quad (27)$$

and  $v_{\varepsilon} \propto v_{pe}$ .

Thus, if the criterion  $v_{pp} \gg v_{pe}$  holds then the symmetric part of the phonon distribution function is the Planck function with the temperature  $T_p$ . This criterion is clear. It means that phonon-phonon collisions occur more frequently than phonon-electron collisions.

Let us obtain now the energy balance equation for electron subsystem. For this purpose, it is necessary to multiply

equation (3) by  $\frac{2\varepsilon}{(2\pi\hbar)^3}$  and integrate it with respect to

electron momentum. As a result, the electron energy balance equation takes the form,

$$\text{div}\vec{Q}_e - \vec{j}\vec{E} = -P_{ep}. \quad (28)$$

Here

$$\vec{Q}_e = -\frac{4}{3(2\pi\hbar)^3} \int_0^{\infty} \varepsilon^2 g(\varepsilon) \vec{V}(\varepsilon) \frac{\partial f_0}{\partial \varepsilon} d\varepsilon \quad (29)$$

is the thermal flux in the electron subsystem,

$$\vec{j} = -\frac{4e}{3(2\pi\hbar)^3} \int_0^{\infty} \varepsilon g(\varepsilon) \vec{V}(\varepsilon) \frac{\partial f_0}{\partial \varepsilon} d\varepsilon, \quad (30)$$

is the electric current, and

$$P_{ep} = -\frac{2}{(2\pi\hbar)^3} \int_0^{\infty} \varepsilon \frac{\partial}{\partial \varepsilon} \left[ T g(\varepsilon) \left( v_{\varepsilon}(\varepsilon) \frac{\partial f_0}{\partial \varepsilon} + \frac{v_{\varepsilon}' f_0 (1-f_0)}{T} \right) \right] d\varepsilon \quad (31)$$

is the term describing the energy transferring from the electrons to phonons. Here  $g(\varepsilon) = 4\sqrt{2\pi m} \varepsilon^{\frac{3}{2}}$  is the density of states,

$$v_{\varepsilon}(\varepsilon) = \frac{1}{2} \pi \sqrt{2mT}^{-1} \varepsilon^{\frac{3}{2}} \int_0^{2\pi} q W(q) \left( N_0 + \frac{1}{2} \right) (\hbar\omega)^2 dq$$

$$, v_{\varepsilon}'(\varepsilon) = \frac{1}{2} \pi \sqrt{2m\varepsilon}^{-\frac{3}{2}} \int_0^{2\pi} q W(q) \hbar\omega dq.$$

Let us note that the item  $\vec{j}\vec{E}$  is the Joule heating and always takes place the following equality,  $P_{ep} = P_{pe}$ .

#### 4. Strong Phonon-Electron Interaction

Let us now consider the situation represented by inequality (22). In this case the phonon-phonon collisions cannot provide a full energetic control in phonon subsystem, and its description becomes more complicated. Since the inequality (22) holds true only for LW-phonons then the parameters which characterize the phonon subsystem essentially depend

on the relationship between  $2\bar{p}$  and  $\frac{T_p}{S}$  [see Eqs. (17)

and (19)]. It should be noted at once that the phonon subsystem can not be characterized by a single temperature in considered case, and the temperature  $T_p$  needs to be specified.

The LW-phonons occupy a small volume in the phase space in comparison with the SW-phonons if Eq. (17) takes place. In this case SW-phonons have enough time to redistribute energy received from LW-phonons among themselves, and the symmetric part of its distribution function is the Planck function with the temperature  $T_p$ . Just

this temperature appears in Eq. (17).

All phonons interact actively with electrons if Eq. (19) holds, and fragmentation all phonons to LW- and SW-phonons have no sense. In this case all phonons are characterized by the temperature  $T_p = T_e$ , where  $T_e$  is the temperature of electrons emitting these phonons, and the inequality  $v_{pe} \gg v_{pp}$  is true for all phonons.

Let us turn to solution of the kinetic equations in the case when conditions (17) and (22) take place.

In the region  $q > 2\bar{p}$  the term  $\frac{1}{4\pi} \int S_{pp} \{N_0 N_0\} d^3\vec{q}$

describes the interaction between the LW-phonons and SW-phonons and the interaction among SW-phonons. It is easy to show that the symmetric part of the distribution function for the momentums  $q > 2\bar{p}$  has the form of the Planck function [see Eq. (23)]. The SW-phonon temperature can be found from the energy balance equation for SW-phonons being similar to Eq.(24) written for LW-phonons. At that, it is necessary to have in mind that the low integration limit over  $\vec{q}$  for  $N_0$  in this case is  $2\bar{p}$  since we are interesting in SW-phonons as a separate subsystem. Calculations lead to that the term containing  $v_{pe}$  is equal to zero, while the term containing  $S_{pp}$  produces the item  $P_{SL}$  determining the energy transfer from the SW- to LW-phonons. As a result, we obtain equation,

$$\text{div}\vec{Q}_p^S = P_{SL}. \quad (32)$$

Since the energy of all phonon subsystem is conserved at phonon-phonon scattering, then the following integral is equal to zero,

$$\frac{1}{(2\pi\hbar)^3} \int (\hbar\omega) S_{pp} d^3\vec{q} = 0 \quad (33)$$

Integration in (33) is made over all values of  $\vec{q}$ , and it can be rewritten as

$$\frac{1}{(2\pi\hbar)^3} \int_{0 < q < 2\bar{p}} (\hbar\omega) S_{pp} d^3\vec{q} = -\frac{1}{(2\pi\hbar)^3} \int_{2\bar{p} < q < \infty} (\hbar\omega) S_{pp} d^3\vec{q} \quad (34)$$

The left-hand side of this equation determines the energy transfer from the LW- to SW-phonons  $P_{LS}$ , and the right-hand of this equation determines the energy transfer from the SW- to LW-phonons. These values are equal,  $P_{LS} = -P_{SL}$ .

The expression for  $\bar{Q}_p^S$  can be obtained with help of Eq. (26), it is only necessary to integrate over  $q$  from  $2\bar{p}$  to  $\infty$ .

In the region  $q < 2\bar{p}$  the term  $\frac{1}{4\pi} \int S_{pp} \{N_0 N_0\} d^3\vec{q}$  describes the interaction of LW-phonons among themselves and with SW-phonons. Nevertheless, one can neglect the contribution of LW-phonon-LW-phonon collisions to the integral  $S_{pp}$  if  $2\bar{p} \ll \frac{T_p}{s}$  because LW-phonons occupy small volume of the phase space. If to take into account that the distribution function of SW-phonons is the Planck function with the temperature  $T_p$ , then the LW-phonon-SW-phonon interaction can be represented as [20]

$$S_{pp} = v_{pp} (N_{T_p} - N), \quad 0 < q < 2\bar{p} \quad (35)$$

where  $v_{pp}(q) = v_{pp}^0 \left(\frac{sq}{T}\right)^t$ . Parameter  $t$  depends on the symmetry of crystal, and  $t = 2$ , for example, for cubic crystals and  $t = 3$  for trigonal crystals. The values  $v_{pp}^0$  can be found in table III of Ref. [4].

It is impossible to represent the phonon-phonon collision integral in the region  $q < 2\bar{p}$  in the form (35) if  $2\bar{p} \gg \frac{T_p}{s}$ . However, in this case all phonons satisfy the condition  $q < 2\bar{p}$ , and as a result inequality  $v_{pe} \gg v_{pp}$  is true for all magnitudes of  $q$ . So, in this case phonon-phonon interaction does not important.

Thus, the kinetic equation (4) for  $N_0$  in the region  $0 < q < 2\bar{p}$  takes the form,

$$-\frac{s}{3} q \frac{\partial}{\partial \vec{r}} \left( \vec{u} \frac{\partial N_0}{\partial (\hbar\omega)} \right) = v_{pe} (N_{T_e} - N_0) + v_{pp} (N_{T_p} - N_0) \quad (36)$$

Eq. (36) can be solved by the perturbation method with respect to parameter  $\frac{v_{pp}}{v_{pe}}$ . In this case we can represent the

symmetric part of phonon distribution function as  $N_0 = N_0^{(0)} + N_0^{(1)}$ , and  $N_0^{(1)} \ll N_0^{(0)}$ .

The zero approximation is

$$N_0^{(0)} = N_{T_e}. \quad (37)$$

The function  $N_0^{(1)}$  determining the second approximation can be found from equation,

$$-\frac{s}{3} q \frac{\partial}{\partial \vec{r}} \left( \vec{u} \frac{\partial N_{T_e}}{\partial (\hbar\omega)} \right) = v_{pp} (N_{T_p} - N_{T_e}) - v_{pe} N_0^{(1)}. \quad (38)$$

Multiplying Eq. (38) by  $\frac{q^2 \hbar\omega}{(2\pi\hbar)^3}$  and integrating over  $\vec{q}$  in the region  $0 < q < 2\bar{p}$ , we can obtain

$$\text{div} \bar{Q}_p^L = P_{pe} + P_{LS}, \quad (39)$$

where

$$P_{LS} = -\frac{1}{(2\pi\hbar)^3} \int_{0 < q < 2\bar{p}} \hbar\omega v_{pp} (N_{T_e} - N_{T_p}) d^3\vec{q}, \quad (40)$$

and

$$P_{pe} = -\frac{1}{(2\pi\hbar)^3} \int_{0 < q < 2\bar{p}} \hbar\omega v_{pe} N_0^{(1)} d^3\vec{q}. \quad (41)$$

Eq. (41) remains indefinite because it contains the unknown value  $N_0^{(1)}$ . Naturally, that the value  $P_{ep}$  containing in Eq. (28) is indefinite too. By this reason, it is necessary to consider the equations (28) and (41) as the set of equations for obtaining both the temperature  $T_e$  and the parameter  $P_{ep}$ .

Eliminating the unknown term  $P_{ep}$  from this system we find equation for obtaining the temperature  $T_e$ ,

$$\text{div}(\bar{Q}_e + \bar{Q}_p^L) - \bar{j}\vec{E} = P_{LS} \quad (42)$$

Thus, electrons and LW-phonons behave themselves as a common subsystem with the temperature  $T_e$  under conditions discussed above. This common subsystem relaxes its energy to SW-phonons with the characteristic rate  $v_{\epsilon} \propto v_{pp}$ .

Electrons and all phonons behave themselves as a common system with the temperature  $T_e$  if  $2\bar{p} \gg \frac{T_p}{s}$ . Let us note only that in this case the term  $P_{LS}$  vanish from Eq.(42) in the process of integrating (34) since this integration is carried out over all phonons, and the total phonon-phonon collisions do not carry out energy from the phonon subsystem.

## 5. Conclusion

In this paper we have considered the energy nonequilibrium electrons and phonons which interact each with other. Nevertheless, as it is known, it is easy to realize the momentum nonequilibrium. This momentum nonequilibrium results in the momentum exchange among charge carriers and phonons in addition to the energy exchange between subsystems. In semiconductors this momentum exchange is known as electron-phonon drag [3, 4].

The electron and phonon nonequilibrium considered in this paper is space-nonuniform in real semiconductors and semiconductor structures. So, this space-nonuniform energy nonequilibrium produces in semiconductors fluxes of different nature. Thus, the energy nonequilibrium has to result in the momentum nonequilibrium of these quasiparticles. The discussion of some aspects of this problem one can find in Refs. [21-25].

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